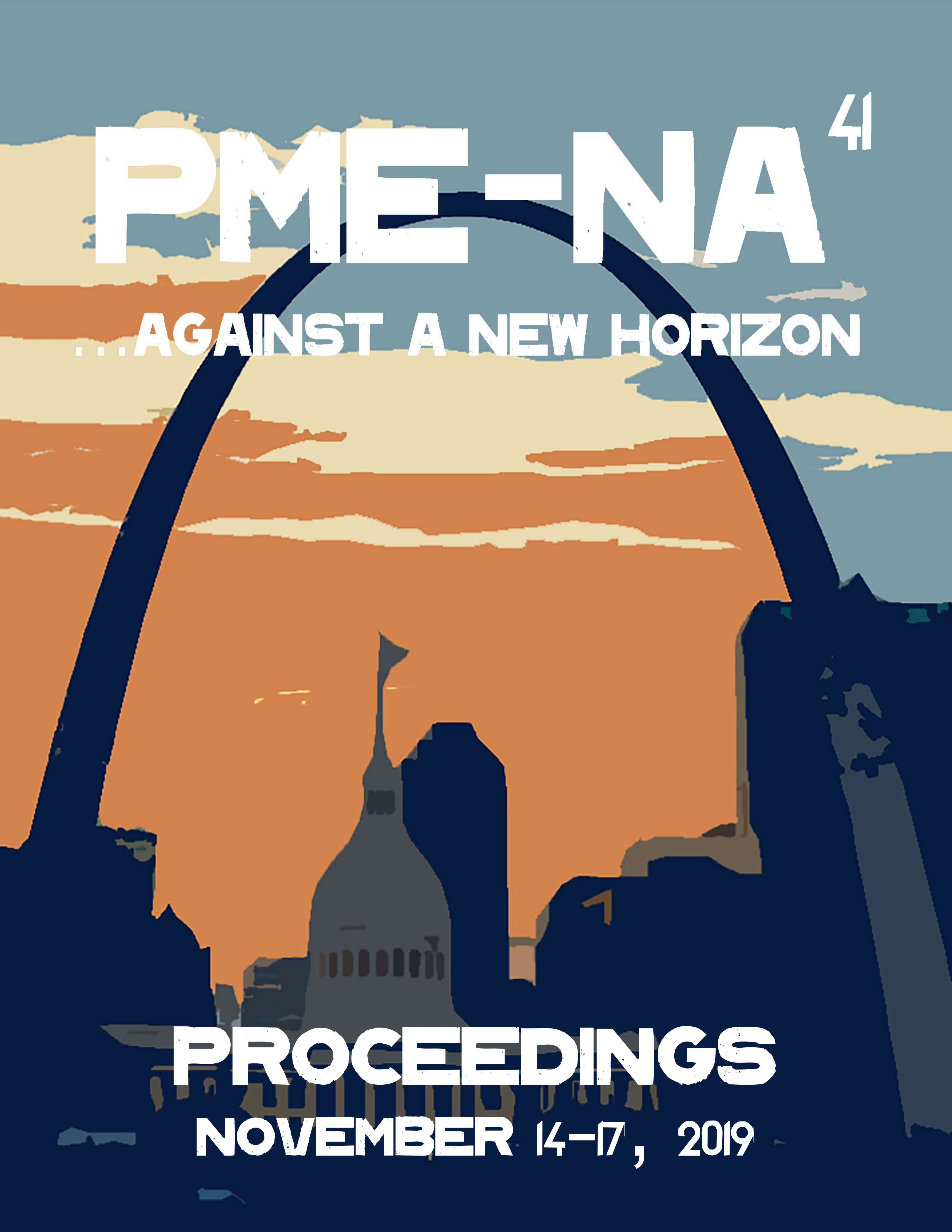

PME-NA 41

...AGAINST A NEW HORIZON

PROCEEDINGS

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THE RELATIONSHIP BETWEEN ANALOGICAL REASONING AND SECOND-GRADERS' STRUCTURAL KNOWLEDGE OF NUMBER

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The aim of the present study was to investigate whether analogical reasoning is predictive of children's structural knowledge of number. A group of 94 second-graders were assessed on their conceptual knowledge of place value and tested on their counting skill, arithmetic fluency, and analogical reasoning. Results showed that the most important predictor of place value knowledge was analogical reasoning after accounting for counting skill and arithmetic fluency. Furthermore, children who were classified as Structural-knowers were more proficient in analogical reasoning than children who viewed numbers on their face value. The results suggest that a structural understanding of number is not only dependent on domain-specific skills, but also on domain-general abilities, including analogical reasoning.

Keywords: Analogical reasoning, Structural knowledge of number, Place value

Understanding in mathematics has been conceptualized as facility with the conceptual structure of a domain (Mason et al., 2009; Richland et al., 2012). One goal of mathematics instruction with young children is to help them acquire a conceptual understanding of number (Mulligan & Mitchelmore, 2009). The decimal numeration system is governed by a multiplicative structure where 10 units in one denomination are grouped, or "bundled," into one unit of the next larger denomination (Ellemor-Collins & Wright, 2009). Knowledge of number structure allows children to use place value concepts to invent meaningful algorithms and develop mental computation skills (Carpenter et al., 1998; Kindrat & Osana, 2018).

Our research focuses on the cognitive processes that are predictive of children's structural knowledge of number and the instructional factors that support such knowledge. We adopted analogical reasoning as a theoretical framework for children's acquisition of number knowledge (English, 2004; Richland, 2011). Analogical reasoning is a cognitive process by which conceptual similarities in two contexts are identified and compared through "structure mapping" (Gentner & Colhoun, 2010). Conceptual understanding can be developed through structure mapping because of the attention to the structural relations between two contexts, such as a written numeral and a pictorial representation of its denominations (English, 2004; Vendetti et al., 2015). In fact, interventions based on instructional analogies have been shown to enhance students' understanding in a number of domains, including fraction division (Sidney & Alibali, 2015), proportionality (Richland & McDonough, 2010), and place value (Mix et al., 2017).

Present Study

The objective of the present study was to test the hypothesis that analogical reasoning is predictive of students' structural knowledge of number after controlling for counting skill and arithmetic fluency, two factors that are known to impact mathematics performance (Geary, 2011; Koponen et al., 2016). The data were part of a larger project investigating the physical affordances of concrete manipulatives on second-graders' numeration knowledge. Outcome measures assessed place-value understanding and number decomposition, administered before

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and after an instructional intervention with manipulatives. Tests of counting skill, arithmetic fluency, and analogical reasoning were administered at pretest. In the present study, we use pretest data on one specific measure of place value, the PicPVT, and examine the extent to which analogical reasoning is predictive of performance on this measure.

Method

Participants

Participants were 94 second-grade children (50 boys; age: $M = 92.3$ months, $SD = 4.8$) from 12 schools in a large, urban area in Canada.

Measures

Place value knowledge. The Picture Place Value Task (PicPVT; Kamawar et al., 2010; Osana & Blondin, 2017) was used to assess whether children understand that a digit in a numeral represents a quantity that is determined by its position. Children were presented with a written numeral with one digit underlined (either the units, tens, or hundreds digit) on a screen. A pictorial representation (i.e., sets of dots) was given below the numeral. By responding "yes" or "no," children indicated to the researcher whether the pictorial representation correctly represented the value of the underlined digit. Of the 20 items, 10 "correct display" items required a positive response from the child (Figure 1a) and 10 "incorrect display" items required a negative response (Figure 1b).

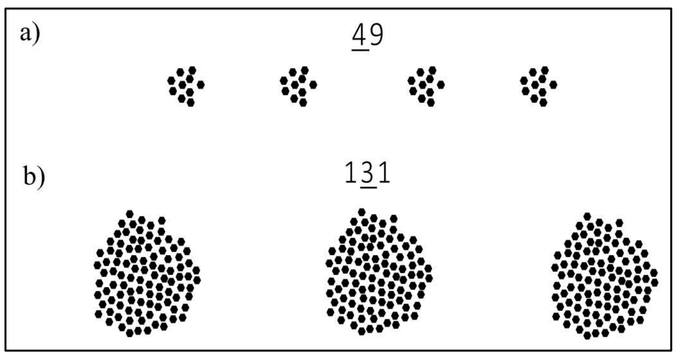

Figure 1: Sample Items on the PicPVT. Panel (a) Shows a Correct Display Item Requiring a Positive Response. Panel (b) Shows an Incorrect Display Item Requiring a Negative Response

Mathematical skills and analogical reasoning. Counting skill was assessed using the Counting and Enumeration task (CE), which required the children to count as high as possible, to skip count by 10 and 100, and to count the number of chips in a collection of 20. Arithmetic fluency was assessed using the Tempo Test Rekenen (TTR, De Vos, 1992), a timed test of mental computation in addition and subtraction. Finally, analogical reasoning was measured using the Raven's Standard Progressive Matrices (Raven et al., 1977). Children chose one picture among four that completed the missing part of a bigger pattern. On all measures, correct responses were assigned 1 point and incorrect responses 0 points.

Results

Descriptive Statistics

The means and standard deviations for all four measures are presented in Table 1 and the zero-order corrections are in Table 2.

Table 1: Means and Standard Deviations for Place Value, Mathematical Skill, and Analogical Reasoning Percent Scores

Measure	*M*	*SD*
PicPVT	.83	.19
Counting and Enumeration (CE)	.75	.19
Arithmetic Fluency (TTR)	.27	.08
Analogical Reasoning (Raven)	.74	.13

Table 2: Zero-order Correlations Between Place Value, Mathematical Skill, and Analogical Reasoning Variables

Measure	1	2	3	4
PicPVT	--	.29**	.25*	.37**
Counting and Enumeration (CE)		--	.33**	.16
Arithmetic Fluency (TTR)			--	.34**
Analogical Reasoning (Raven)				--

*p<.05, **p<.01

Predictors of Place Value Knowledge

We conducted a hierarchical multiple regression to test the relationship between place value knowledge and analogical reasoning. The criterion was PicPVT score. The first two predictors, counting skill and arithmetic fluency, were entered in the first and second steps in the analysis, and were entered together because they were similarly correlated with place value knowledge. In the third step, analogical reasoning was entered separately to allow for an independent assessment of the variance explained by analogical reasoning. The regression statistics are reported in Table 3.

Table 3: Summary of Hierarchical Regression Analysis for Variables Predicting Place Value Knowledge

Variable	R	R^2	R^2 change	Final β	*t*
Step 1	.33	.11	.11		
Counting and Enumeration (CE)				.24	2.24*
Arithmetic Fluency (TTR)				.16	1.52
Step 2	.44	.19	.08		
Counting and Enumeration				.22	2.15*
Arithmetic Fluency (TTR)				.07	0.64
Analogical Reasoning (Raven)				.30	3.01**

*p<.05, **p<.01

At Step 1, the model explained a significant 10.7% of the variance in place value knowledge, $F(2, 90) = 5.37$, $p < .01$. Introducing analogical reasoning in the model at Step 2 explained an additional 8.3% of the variance in place value knowledge, and this change was significant, $F(1, 89) = 9.07$, $p < .01$. Together, the three predictors in the model explained 19% of the variance in place value knowledge. In the final model, arithmetic fluency was not related to place value knowledge. The most important predictor was analogical reasoning, $\boldsymbol{\beta} = .30$, $t(89)=3.01$, $p < .01$.

In a second analysis, we created profiles of place value knowledge by identifying patterns of responses on the correct display and incorrect display items on the PicPVT. Children who correctly answered more than five incorrect display items and more than five correct display items were placed in the Structural Knowledge (SK) profile ($n = 74$). Children in the SK profile understood that a digit represents a quantity depending on its position in a numeral. Children who correctly answered fewer than 4 incorrect display items and more than 5 correct display items likely counted the number of groups, regardless of the number in each group. We argue that these children held a "concatenated digits" view of number (Bergeron & Herscovics, 1990), seeing numerals at their "face value" (Barnett-Clarke et al., 2010). We placed these children in a Face Value (FV) profile ($n = 16$). Finally, four children displayed no discernable patterns of responses and were likely guessing. We removed these four from the analysis.

A one-way ANCOVA was performed with profile as the independent measure (SK, FV) and analogical reasoning as the dependent measure. Counting skill and arithmetic fluency were entered as covariates. A significant effect of profile was found, $F(1, 85) = 198.32$, $p < .001$, partial $\eta^2 = .7$, with higher analogical reasoning scores in the SK profile ($M = .91$, $SD = .10$) than in the FV profile ($M = .53$, $SD = .08$).

Discussion

The findings suggest that analogical reasoning is predictive of children's number knowledge, even when accounting for counting and arithmetic skills. This suggests that a structural understanding of number is not only dependent on domain-specific skills, such counting skill and prior mathematical knowledge (Fyfe et al., 2012; Zhang et al., 2014), but also on domain-general abilities, including analogical reasoning (Geary et al., 2017). Our research thus contributes to the literature on the cognitive predictors of children's structural knowledge of number, a critical element to children's success in mathematics, beginning in the early years (Byrge et al., 2014). In particular, our findings align with those of Collins and Laski (2019), who showed that relational reasoning is predictive of numeral-quantity knowledge and magnitude comparison.

One discernable pedagogical implication that emerges from our research is that educators may wish to focus on developing young children's analogical reasoning skill because of its relationship to structural knowledge of mathematics. Although our research was correlational, the findings allow us to predict that enhancing children's analogical reasoning would increase their conceptual understanding of number and reduce the reliance on superficial aspects of mathematical representations.

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